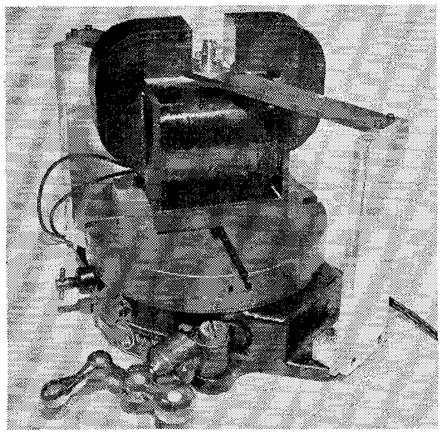

Fig. 1—Device for orienting ferromagnetic single crystals using rotating electromagnet.

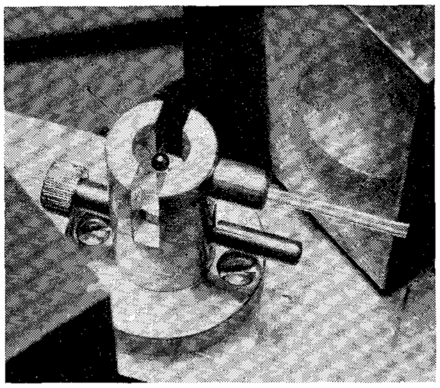

Fig. 2—Aligning jig showing YIG sphere attached to wire along one easy axis and quartz rod along [110] axis.

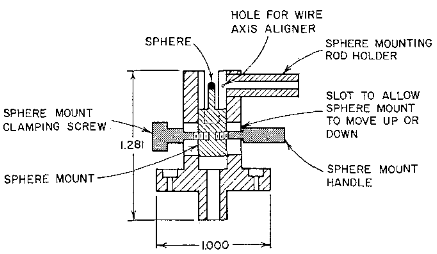

Fig. 3—Construction of aligning jig for mounting crystal along [110] axis.

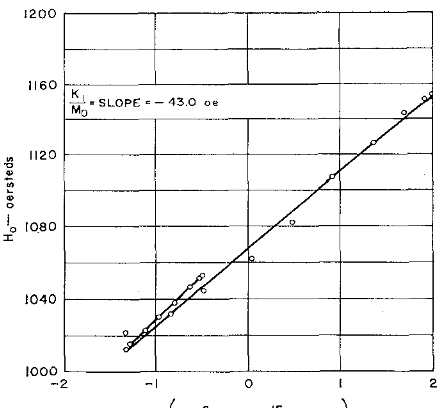

Fig. 4—Measurement of first-order anisotropy constant of yttrium-iron-garnet at $T=25.6°C$.

a [100] axis of the sample and the dc magnetic field. These measurements were performed at $f_0=3$ Gc using a crossed-strip-line coupler that is similar to the crossed-guide coupler which was described elsewhere.[4] The value of K_1/M_0, which was determined from the slope of this plot, is equal to −43.0 oersteds measured at an ambient room temperature of 25.6°C. This value of K_1/M_0 agrees very well with the value measured by Dillon,[5] $K_1/M_0 \cong -42.5$ oersteds; very well with the value of $K_1/M_0 = -43.2 \pm 0.1$ oersteds measured by Hill and Bergman;[6] not as well with the value $K_1/M_0 \sim -25$ oersteds reported by Rodrique, *et al.*,[7] and fairly well with the value of $K_1/M_0 = -35$ oersteds reported by Czerlinsky and Field.[8]

We have also measured the anisotropy constant K_1/M_0 of 600 and 950 gauss gallium-substituted yttrium-iron-garnet.[9] The results of these measurements are shown in Table I below.

TABLE I
ANISOTROPY CONSTANTS OF GALLIUM-SUBSTITUTED YTTRIUM-IRON-GARNET

M_0	T	K_1/M_0
600 gauss	22.9°C	−55.8 oersteds
950±50 gauss	25°C	−41.7 oersteds

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[4] P. S. Carter, Jr., "Design Criteria for Microwave Filters and Coupling Structures," Stanford Res. Inst., Menlo Park, Calif., Tech. Rept. 8, Contract DA-36-039, SC-74862, SRI Project 2326; October, 1959.

[5] J. F. Dillon, "Ferrimagnetic resonance in yttrium iron-garnet," *Phys. Rev.*, vol. 105, pp. 759–760; January 15, 1957.

[6] R. M. Hill and R. S. Bergman, "Nonlinear response of YIG," *J. Appl. Phys.* vol. 325, pp. 227–228; March, 1961.

[7] G. P. Rodrique, H. Meyer and R. V. Jones, "Resonance measurements in magnetic garnets," *J. Appl. Phys.* vol. 315, pp. 376–382; May, 1960.

[8] E. R. Czerlinsky and W. G. Field, "Magnetic Properties of Ferrimagnetic Garnet Single Crystals," Electronic Material Sciences Lab., Electronics Res. Directorate, Air Force Cambridge Res. Ctr., L. G. Hanscom Field, Bedford, Mass. (Unpublished.)

[9] These materials were purchased from Microwave Chemical Co., New York, N. Y.

Millimeter Wave Cavity Coupling by Quarter-Wave Transformer*

INTRODUCTION

In order to use high mode-order microwave cavities (Fabry-Perot,[1] confocal,[2] or biconical spherical[3] resonators) to achieve either high field intensities or high selectivity with low transmission losses, it is necessary to couple power into the resonators efficiently. This may be accomplished if the coupling system acts as a transformer matching the lossy elements of the cavity to a source of input power, and if the coupling system has an aperture large enough to make diffraction losses negligible.

This note describes an easily designed coupler which satisfies these requirements. It is essentially a series iteration of parallel plane waveguides, each λ/4 long. The fundamental design principles of this coupler are described and applied to the particular case of a bisected confocal resonator. Experimental evaluation verifies the efficacy of both the transformer and the approximations used in its design.

PRINCIPLES OF TRANSFORMER DESIGN

The essence of electromagnetic cavity coupling is the establishment of a balance between the power passing inward through the coupling aperture and that power lost to the cavity walls and its filling materials. Thus a cavity of volume V enclosed by the lossy surface S and coupled by the aperture A satisfies the following balance:

$$\tfrac{1}{2}\,\mathrm{Re}\int_A \boldsymbol{E}_A \times \boldsymbol{H}_A{}^* \cdot d\boldsymbol{A} = \tfrac{1}{2}\,\mathrm{Re}\int_S \boldsymbol{E}\times\boldsymbol{H}^*\cdot d\boldsymbol{S} + \tfrac{1}{2}\,\mathrm{Re}\int_V \sigma\boldsymbol{E}\cdot\boldsymbol{E}^* dV. \quad (1)$$

Let consideration be given just to the condition of resonance, when $\boldsymbol{E}_A \times \boldsymbol{H}_A{}^*$ is purely real. Since the coupler will have a very low impedance and will be considered essentially as a perturbation, let the field amplitudes within the cavity be characterized by $\boldsymbol{H}_A$, the average magnetic field at the surface of the cavity in the aperture region. The definition of K, the total loss factor, is

$$K\boldsymbol{H}_A\cdot\boldsymbol{H}_A{}^* = \mathrm{Re}\int_S \boldsymbol{E}\times\boldsymbol{H}^*\cdot d\boldsymbol{S} + \mathrm{Re}\int_V \sigma\boldsymbol{E}\cdot\boldsymbol{E}^* dV. \quad (2)$$

In most cases, this quantity includes losses due to

1) Finite surface resistance of metallic walls.
2) Diffraction at the ends of an open cavity.
3) Any coupling apertures other than the one under consideration.
4) Losses coupled by the media filling the cavity.

The last category should include crystals, plasmas or any other system driven by the cavity fields.

The coupling electric field $\boldsymbol{E}_A$, assumed to be perpendicular to $\boldsymbol{H}_A$, is regarded here as a perturbation only. If the fields across the aperture are approximately constant, its magnitude must satisfy the following power balance:

$$\tfrac{1}{2}E_A H_A{}^* A = \tfrac{1}{2}KH_A H_A{}^*. \quad (3)$$

* Received July 20, 1962. This work was supported by the Air Force Office of Scientific Research under Grant AF 62-287.

[1] W. Culshaw, "High resolution millimeter wave Fabry-Perot interferometer," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 182–189; March, 1960.

[2] G. D. Boyd and J. P. Gordon, "Confocal multimode resonator for millimeter through optical wavelengths," *Bell Sys. Tech. J.*, vol. 40, pp. 489–508; March, 1961.

[3] W. Culshaw, "Resonators for millimeter and submillimeter wavelengths," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-9, pp. 135–144; March, 1961.

This relation defines the input wave impedance Z_A in the aperture.

$$Z_A = \frac{E_A}{H_A} = \frac{K}{A}. \tag{4}$$

This wave impedance is of the order of 0.1 to 10 ohms in typical cases, and it is the impedance which must be transformed to an input impedance βZ_0. The coefficient β is the conventional coupling coefficient, and Z_0 is the wave impedance of the input wave, typically around 377 ohms.

The necessary transformation may be made with a quarter-wave transmission line of characteristic impedance Z_T, which possesses the well-known transformation property

$$Z_{IN} Z_{OUT} = Z_T^2. \tag{5}$$

This Z_T must be the characteristic impedance presented to a quasi-plane wave over a large aperture. Fig. 1 shows the structure of a $\lambda/4$ transformer whose plane wave characteristic impedance may be tailored to the required values. Each slit of height a behaves as a parallel plane waveguide propagating a wave that is approximately TEM. As long as the spacing b of these guides is less than a wavelength in the media adjacent to the transformer, there will be no grating diffraction. The plane wave characteristics are determined by the averages of E and H across the transformer surface; consequently, the characteristic impedance Z_T is

$$Z_T = \frac{E_{AVG}}{H_{AVG}} = \frac{a}{b}\frac{E_{TEM}}{H_{TEM}} = \frac{a}{b}\left(\frac{\mu_T}{\epsilon_T}\right)^{1/2}. \tag{6}$$

This presumes the guides are filled with a low-loss material described by μ_T and ϵ_T.

Drawing (4), (5), (6) and the $\lambda/4$ condition together, a transformer design may be specified by the specification of Z_0, β, f (operating frequency), A and a knowledge of the cavity losses.

$$\frac{a}{b} = \left(\frac{\beta Z_0 K/A}{\mu_T/\epsilon_T}\right)^{1/2}. \tag{7}$$

$$d = \tfrac{1}{4}(\epsilon_T \mu_T f^2)^{-1/2}. \tag{8}$$

The slot characteristics are specified only by their ratio, but the choice of b is limited by the condition that it be less than λ in the exterior media.

Since the design of the coupling system is based upon cavity losses, its accuracy depends directly upon the accuracy with which those losses may be anticipated.

The Fabry-Perot and Confocal Resonators

By defining an effective aperture for the confocal resonator, it may be handled with the simplicity of an idealized Fabry-Perot resonator, particularly if its reflectors are large enough to make diffraction losses negligible. Physically, coordination between the fields in the cavity and the driving field, with respect to both aperture size and equiphase surfaces, is implicit but necessary.

Neglecting diffraction losses, an empty cavity has loss due to surface resistance; if R_T and R_A are the surface resistances of the two ends of the cavity, the loss relation is

$$\tfrac{1}{2}K\,|H_A|^2 = \frac{R_A + R_T}{2}A\,|H_A|^2, \tag{9}$$

assuming that the entire cavity end (or effective aperture) is to be used as a coupling surface. From Boyd and Gordon,[2] the effective aperture is found to be $\rho\lambda$, the product of the radius of curvature of the reflectors and the operating wavelength.

It follows directly from (9) that

$$Z_A = \frac{K}{A} = R_A + R_T. \tag{10}$$

Consequently,

$$Z_T = [\beta Z_0(R_A + R_T)]^{1/2} = \frac{a}{b}\left(\frac{\mu_T}{\epsilon_T}\right)^{1/2}. \tag{11}$$

The properties of these cavities that are commonly of practical interest are the quality factor Q and the shunt resistance R_S. These are readily derived from either the field relations or the transmission line analog frequently used for nearly plane waves.

$$Q_L = \frac{n\pi}{2}\frac{Z_D}{(1+\beta)(R_A + R_T)}. \tag{12}$$

$$R_S = \frac{|E_{MAX}|^2 A}{2P_{INC}} = \frac{4\beta}{(1+\beta)^2}\frac{Z_D^2}{R_T + R_A}. \tag{13}$$

These formulations apply to a cavity containing a lossless dielectric with intrinsic impedance Z_D, which is 377 ohms for the empty cavities. The shunt resistance definition is based upon the power incident on the cavity.

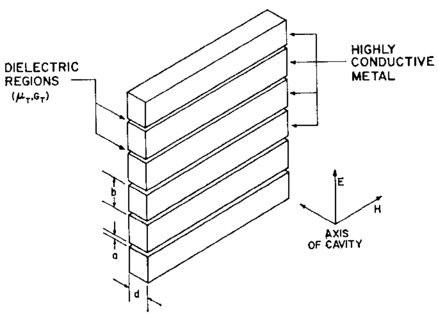

Fig. 1—The structure of an iterated parallel plane waveguide transformer.

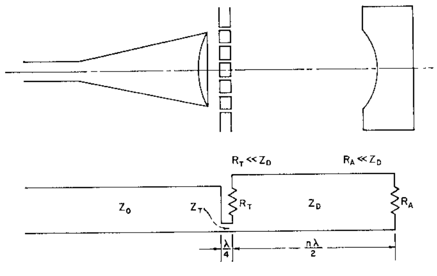

Fig. 2—A diagram of the bisected confocal resonator coupled by a $\lambda/4$ transformer, with its approximate transmission line equivalent circuit.

Experimental Evaluation

The behavior of the transformer coupler was determined in a bisected confocal resonator coupled by a lens corrected horn. Two different transformers were tested in this configuration; one was designed for 35 kMc and the other for 70 kMc. The system is diagrammed in Fig. 2, together with its simplified equivalent circuit.

Both the transformer and the confocal reflector were brass, and the coupling parameters were based upon the ideal surface resistivities of brass. The effective aperture of the system was 5 cm², and the horn diameter was selected to provide a good match between the TE_{11} field distribution in the circular horn and the approximately Gaussian field distribution in the cavity itself. Failure to do this caused a serious lowering of the cavity Q.

In summary, the measurements were as follows:

- Q_L The loaded Q was determined by measuring the frequency difference between half-power points of the resonances at both 35 kMc and 70 kMc.
- β The coupling coefficient was determined by VSWR measurements at 35 kMc and by hybrid-T bridge measurements at 70 kMc.
- R_S The shunt resistance was determined by observing the heating of a bead thermistor placed in the cavity with its leads parallel to the H field. The 0.013-inch diameter thermistor was calibrated in a matched guide.

A knowledge of β, Q_L, and the source impedance Z_0 is sufficient to determine both the apparent characteristic impedance of the transformer and the total losses of the cavity.

The experimental data is summarized in Tables I and II.

TABLE I

35 kMc Evaluation

Transformer Construction
$a = 0.005$ inch $d = 0.059$ inch
$b = 0.154$ inch $\epsilon_T = 2.08\epsilon_0$ (Teflon)

Characteristic		Ideal	Observed
n	Mode number	5	5
Z_0	Source impedance	377 Ω	320 Ω
$R_A + R_T$	Cavity losses	0.192 Ω	0.24 Ω
Z_T	Transformer impedance	8 54 Ω	8.7 Ω
Q_L	Loaded Q	7700	6300
β	Coupling coefficient	1	1
R_S	Shunt resistance	7×10^5 Ω	5×10^5 Ω

TABLE II

70 kMc Evaluation

Transformer Construction
$a = 0$ 004 inch $d = 0.041$ inch
$b = 0$ 148 inch $\epsilon_T = \epsilon_0$

Characteristic		Ideal	Observed
n	Mode order	10	10
$R_A + R_T$	Cavity losses	0.273 Ω	0.43 Ω
Z_T	Transformer impedance	10.2 Ω	9.9 Ω
Q_L	Loaded Q	10,900	8000
β	Coupling coefficient	1	0.55

Two items not specifically evaluated should be noted. The transformers tested were not particularly frequency sensitive; they behaved effectively over approximately a 20 per cent range. The several transformers tested thus far have indicated no important dependence upon a and b; only their ratio is important.

Conclusion

The utilization of an iteration of parallel plane waveguides as a $\lambda/4$ transformer to couple a millimeter wave cavity has been shown to be both effective and simple to design. Its construction is also straightforward. The accuracy of the design procedure, though based upon the use of average fields and a perturbation impedance calculation, is limited mainly by the inaccuracies in anticipated losses.

By field averaging, other waveguide impedance transforming schemes should be adaptable to the coupling of high order microwave cavities. Another variation of the $\lambda/4$ transmission line is a simple dielectric sheet. Available dielectrics limit this scheme to use in rather lossy cavities, however.

Systems which use wave structures above cutoff as transformers overcome the difficulties of achieving critical coupling experienced with reactive reflectors in lossy cavities.

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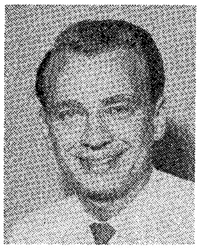

From 1952 to 1956 he served as an Electronic Technician with the U. S. Air Force. Also, prior to college, he was employed as an Electronic Technician with the Hazeltine Electronics Company, Little Neck, N. Y. From 1958 to 1962 he was a Standards Engineer at Hughes Aircraft Company, Culver City, Calif. He is presently a Research Assistant at the Aerospace Corporation, El Segundo, Calif.

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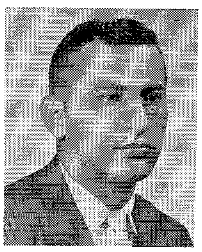

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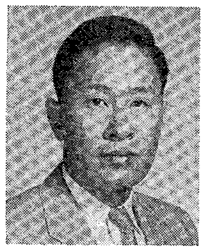

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